

A GUIDE TO HELP TEACHER'S UNDERSTAND COGNITIVELY GUIDED INSTRUCTION



Cognitively Guided Instruction

Cognitively Guided Instruction is a primary level approach to teaching mathematics that was developed at the Wisconsin Center for Education Research, Madison, Wisconsin. This extensively researched approach (Carpenter et al., 1999) strives to provide teachers with opportunities to understand how young children develop mathematical thinking. Possessing this knowledge enables teachers to plan instruction to guide students toward learning important mathematical skills - with understanding.

Developing Mathematical Reasoning and Whole-Number Knowledge Using Word Problems

Cognitively Guided Instruction uses word problems to develop mathematical thinking. These problems are based on detailed analyses of various problem situations involving number. The word problems are organized within a framework (see chart below) that makes it possible to distinguish among problems in terms of difficulty. This knowledge enables teachers to sequence word problems from easiest to most difficult. Such sequencing promotes the development of mathematical reasoning.

The problem chart (below) provides examples of 14 different types of word problems. Number sentences have been included to help teachers identify the missing quantity and determine the relative difficulty among problems. The problems are coded according to difficulty level.

An 18" x 24" colored chart is available at:

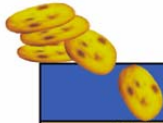
<http://www.ritesandrightrep.com/Pages/TaiwanResources.htm>

CGI Resource Information

A text describing this approach, Children's Mathematics: Cognitively Guided Instruction, is available at: <http://www.heinemann.com>

CGI website: <http://www.ritesandrightrep.com/Pages/CGI.htm>

Taiwan's Cookie Problems



Joining Problems		
Join: Result Unknown (JRU)	Join: Change Unknown (JCU)	Join: Start Unknown (JSU)
<p>♣ Bear Dog had 5 cookies. Taiwan gave him 8 more cookies. How many cookies does Bear have now?</p> <p>$5 + 8 = \square$</p>	<p>♦ Bear Dog had 5 cookies. Taiwan gave him some more. Then Bear Dog had 13 cookies. How many cookies did Taiwan give Bear Dog?</p> <p>$5 + \square = 13$</p>	<p>♥ Bear Dog had some cookies. Taiwan gave him 8 more. Then he had 13 cookies. How many cookies did Bear Dog have before Taiwan gave him any?</p> <p>$\square + 8 = 13$</p>
Separating Problems		
Separate: Result Unknown (SRU)	Separate: Change Unknown (SCU)	Separate: Start Unknown (SSU)
<p>♣ Taiwan had 13 cookies. He gave 5 of them to Bear Dog. Then how many cookies did Taiwan have?</p> <p>$13 - 5 = \square$</p>	<p>♦ Taiwan had 13 cookies. He gave some to Bear Dog. Now he has 5 cookies left. How many cookies did Taiwan give Bear Dog?</p> <p>$13 - \square = 5$</p>	<p>♥ Taiwan had some cookies. He gave 5 to Bear Dog. Now he has 8 cookies left. How many cookies did Taiwan have to start with?</p> <p>$\square - 5 = 8$</p>
Part - Part - Whole Problems		
Part Part Whole: Whole Unknown (PPW:WU)		Part Part Whole: Part Unknown (PPW:PU)
<p>♣ Bear Dog has 5 chocolate chip cookies and 8 peanut butter cookies. How many cookies does Bear Dog have altogether?</p> <p>$5 + 8 = \square$</p>		<p>♦ Bear Dog has 13 cookies. Five are chocolate chip and the rest are peanut butter. How many peanut butter cookies does Bear Dog have?</p> <p>$13 - 5 = \square$ $5 + \square = 13$</p>
Comparing Problems		
Compare: Difference Unknown (CDU)	Compare: Quantity Unknown (CQU)	Compare: Referent Unknown (CRU)
<p>♣♦ Taiwan has 8 cookies. Bear Dog has 5 cookies. How many more cookies does Taiwan have than Bear Dog?</p> <p>$8 - 5 = \square$ $5 + \square = 8$</p>	<p>♥ Bear Dog has 5 cookies. Taiwan has 3 more cookies than Bear Dog. How many cookies does Taiwan have?</p> <p>$5 + 3 = \square$</p>	<p>♥ Taiwan has 8 cookies. He has 3 more cookies than Bear Dog. How many cookies does Bear Dog have?</p> <p>$8 - 3 = \square$</p>
Multiplying and Dividing Problems		
Multiplication	Measurement Division	Partitive Division
<p>♣ Bear Dog has 4 piles of cookies. There are 3 cookies in each pile. How many cookies does Bear Dog have?</p> <p>$4 \times 3 = \square$</p>	<p>♣♦ Taiwan has 12 cookies. If he puts 3 cookies in each bag, how many bags will he fill?</p> <p>$12 \div 3 = \square$</p>	<p>♣♦ Taiwan has 12 cookies. He wants to give them to 3 friends. If he gives the same number of cookies to each friend, how many cookies will each friend get?</p> <p>$12 \div 3 = \square$</p>



Difficulty Levels
 Level I ♣
 Level II ♣♦
 Level III ♦♦
 Level IV ♥



Building on Children’s Intuitive Understanding

A goal of Cognitively Guided Instruction is that young children become independent problem solvers, students who are able to approach and solve word problems without having to rely on the teacher’s telling them how to do it.

A number of factors influence whether a problem is more or less difficult for a child to solve independently. Understanding these factors will help the teacher decide on the word problems to be used during instruction. These factors include the following:

Whether the Problem Involves an Action Situation that the Child Can Act Out

A problem that can be acted out is easier for a child to solve than one that cannot be acted out. For example, the first of the following two problems is easier. Here a child can actually pretend to give the cookies away. The second problem is more difficult because the problem cannot be acted out directly.

SRU Problem (action direct):

Taiwan had 8 cookies. He gave 3 of them to Bear.
How many cookies does Taiwan have now?

SRU Problem (action indirect):

Taiwan gave 3 cookies to Bear. He started with 8 cookies.
How many cookies does Taiwan have now?

Whether a Problem Can Be Modeled or Acted Out in the Order in Which It Is Heard

When first learning to solve word problems, young children approach them in the order in which they hear them. They do not begin at the end of the problem and work backward. The ability to do this develops after children have had many experiences with solving problems and have developed an understanding of the relationships among the numbers within a problem. For example, they understand that combined parts make up the total. For this reason, the first of the following two problems is easier. The wording encourages the child to set out five counters and then add eight more. However, the second problem does not specify a beginning number. The young child who has not yet developed the ability to relate a part of a quantity to the total quantity will respond to the second question with, “Some. Bear had some cookies.”

JRU Problem (can be solved in order given):

Bear had 5 cookies. Taiwan gave him 8 more.
How many cookies does Bear have now?

JRU Problem (cannot be solved in order given):

Bear had some cookies. Taiwan gave him 8 more. Then she had 13 cookies. How many cookies did Bear have before Taiwan gave him any?

Whether the Problem Can Be Directly Modeled

When all the quantities given in a problem refer to complete sets of physical objects or amounts, the problem can be modeled directly. A word problem that can be directly modeled, that is, represented in some concrete way (on fingers, by manipulating counters, with tally marks or drawings), is easier.

The first of the following two problems is easier because each quantity can be set out or modeled directly. When modeling this problem with counters, a young child will count out the two quantities, lining them up side by side, and match the rows to determine the difference.

In the second problem, the 3 does not refer to a quantity that exists in the situation. That is, neither Taiwan or Bear has 3 cookies. So, the 3 cannot be set out or modeled directly when setting up the problem situation. To solve this type of problem, the child must have developed the ability to mentally visualize and analyze the problem.

CDU Problem (direct modeling situation):

Taiwan has 8 cookies. Bear has 5 cookies.

How many more cookies does Taiwan have?

CRU Problem (cannot be modeled, directly analysis required):

Bear has 5 cookies. She has 3 fewer cookies than Taiwan.

How many cookies does Taiwan have?

Whether the Unknown Quantity Is Located at the End, Middle, or Beginning of the Problem

Because young children solve problems in the sequence that they hear them, problems that are worded in such a way so that the unknown quantity is located at the end (first example below) are easier to solve. Problems with the missing quantity in the middle (second example below) or at the beginning (third example below) are more difficult.

As the child's understanding of quantity and relationships among quantities develops, the child will be able to make sense of the entire problem, represent the situation, and plan a solution. When a child is able to do these steps, the child will not need to use manipulatives. Rather, the child will mentally manipulate quantities.

SRU Problem (location of unknown at end of problem):

Bear had 7 cookies. She gave 4 to Taiwan. How many cookies does Bear have now?

SCU Problem (location of unknown in middle of problem):

Taiwan had 8 cookies. He gave some to Bear. Now he has 5 cookies.

How many cookies did Taiwan give to Bear?

SSU Problem (location of unknown at start of problem):

Taiwan had some cookies. He gave 3 cookies to Bear.

Then he had 5 cookies left.

How many cookies did Taiwan have before sharing with Bear?

Simple Multiplication or Division Problems that Can Be Acted Out or Modeled

Very young children can solve small number multiplication and division problems because such problems can be easily modeled. However, it is important that children first solve many problems involving joining and separating situations. Experiences solving joining and separating problems will help the child focus on what is being asked - solving simple problems allows them to develop the ability to think about numerical quantities within the context of situations and to make sense of the question being asked. Once children have learned to focus on the question being asked, they are ready for simple multiplication and division problems.

Multiplication problem:

Bear has 4 piles of cookies. There are 3 cookies in each pile. How many cookies does Bear have?

Measurement Division problem:

Bear gave 12 cookies to some children. She gave each child 3 cookies. How many children got cookies?

Partitive Division problem:

If Taiwan gives 12 cookies to 3 friends and each friend is given the same number, how many cookies will each friend get?

Developmental Levels of Children's Thinking Revealed in Intuitive Solution Strategies

The extensive CGI research documented the developmental thinking processes that children go through when learning to solve word problems. It is important to emphasize that these processes are intuitive, ones that are not taught to the student by a teacher.

To effectively promote students' learning, a teacher must clearly understand the relationships among the different types of word problems discussed in the previous sections and the developmental levels of children's thinking. Most children pass through three levels when acquiring problem-solving skills. Initially, they solve problems using modeling strategies. Over time, modeling strategies are replaced by counting on or back strategies. Finally, most children come to rely on number facts. Detailed descriptions of how children's solutions vary depending on the problem that they are solving and their developmental ability are provided in the following sections.

Relating Solution Strategies to Developmental Stages

Level 1: Examples Involving Direct Modeling

A child using a Direct Modeling strategy represents each set of numbers with concrete objects. In the following examples, the child models with counters.

Join: Result Unknown (JRU)

Taiwan had 6 cookies.
Bear gave him 5 more.
How many cookies does
Taiwan have now?

**Child’s Direct Modeling Strategy
for JRU (Above)**

“Taiwan had 6 cookies . . .
one, two, three, four, five, six.”
(Child sets out 6 counters.)
“Bear gave him five more . . .
one, two, three, four five.”
*(Child sets out 5 counters, then
pushes both sets together and counts all.)*
“He has eleven cookies.”

Separating: Result Unknown (SRU)

Bear had 11 cookies.
She gave 5 to Taiwan.
How many cookies does
Bear have now?

**Child’s Direct Modeling Strategy
for SRU (Above)**

“Bear had 11 cookies . . .
one, two, three, four, five, six, seven,
eight, nine, ten, eleven.” *(Child sets out 11
counters.)* “She gave five . . . one, two
three, four, five.” *(Child removes 5 counters
from the 11 and counts remaining counters.)*
“Now she has . . . one, two, three, four, five,
six. She has six.”

Level 2: Examples Involving Counting Strategies

A child using a counting strategy holds a number in him/his mind and counts on or back from that number keeping track of the quantity of objects added on or removed with fingers or tally marks.

A child at this level must be able to immediately recognize amounts modeled on fingers without having to recount the fingers (group recognition). Group recognition is required so that the child knows when to stop counting on or back.

Join: Result Unknown (JRU)

Taiwan had 6 cookies.
Bear gave him 5 more.
How many cookies does
Taiwan have now?

**Child’s Counting Strategy to Solve
JRU Problem (above)**

“I don’t have to count the six again.
I just have to add five to it.
I say , ‘seven, eight,
nine, ten, eleven’ .” *(Child holds
up a finger with each count.)*
“I have eleven.”

Separating: Result Unknown (SRU)

Bear had 11 cookies.
She gave 5 to Taiwan.
How many cookies does
Bear have now?

**Child’s Counting Strategy to
SRU (above)**

“I know Bear had eleven cookies.
I know she gave five away. So I count
five down from eleven.
‘Eleven, ten, nine, eight, seven.’
I have six left.”
(Child folds a finger down with each count.)

Level 3: Examples Involving Deriving Strategies

A child possessing reasoning ability and good number sense is able to solve problems in flexible ways, often breaking numbers down and recombining them by using known facts. This child frequently visualizes the quantities and solves the problem with mental math.

Join: Result Unknown (JRU)

Taiwan had 6 cookies.
Bear gave him 5 more.
How many cookies does
Taiwan have now?

Separating: Result Unknown (SRU)

Bear had 11 cookies.
She gave 5 to Taiwan.
How many cookies does
Bear have now?

Child's Deriving Strategy to Solve the JRU Problem (above)

"I know that five and five is ten.
I took one from the six to make five,
so I have to add the one back on.

It's eleven."

Child's Deriving Strategy to Solve the SRU Problem (above)

"I know that ten take away five is five,
but I started with eleven. The answer
must be one more. It's six."

Matching Solution Strategies to Problem Situations

The actions and decisions that a child makes when solving a problem are determined by the problem posed to the child and the child's level of development. Each of the 14 different problem situations requires different reasoning processes. Examples are given below.

Strategies Used by Level 1 Children (Direct Modelers) for Specific Problem Situations

At this level, the child concretely represents (using counters, fingers, tally marks, drawings) all numerical quantities within the problem.

JRU Problem

Taiwan had 3 cookies. Bear gave him 5 more. How many cookies does Taiwan have now?

Joining-All Solution:

The child constructs (with manipulatives) a set of three objects and a set of five objects. The child pushes the sets together and the union of the two combined sets is counted.

JCU Problem

Bear has 5 cookies. Taiwan gave him some more cookies.
Now Bear has 8 cookies.
How many cookies did Taiwan give him?

Joining-To Solution:

The child constructs (with manipulatives) a set of three objects. Objects are added to this set until there is a total of eight objects. The child counts the number of objects that were added to find the answer.

SRU Problem

Taiwan had 8 cookies. He gave 3 to Bear.
How many cookies does Taiwan have now?

Separating-From Solution:

The child constructs (with manipulatives) a set of eight objects. Three objects are removed. The answer is the number of remaining objects.

SCU Problem

Bear had 8 cookies. She gave some to Taiwan.
Now Bear has 3 cookies.
How many cookies did he give to Taiwan?

Joining-To Solution:

A set of eight objects is counted out. Objects are removed from it until the number of objects remaining is equal to three. The answer is the number of objects removed.

CDU Problem

Taiwan has 3 cookies. Bear has 5 cookies.
How many more cookies does Bear have than Taiwan?

Matching Solution:

A set of three objects and a set of eight objects are matched one to one until one set is used up. The answer is the number of unmatched objects remaining in the larger set.

JSU Problem

Bear had some cookies. Taiwan gave him 3 more.
Now he has 8 cookies.
How many cookies did Bear have to start with?

Trial-and-Error Solution:

A set of objects is constructed. A set of three objects is added to or removed, and the resulting set is counted. If the final count is eight, then the number of objects in the initial set is the answer. If it is not right, then a different initial set is tried.

**Strategies Used by Level 2 Children (Counting On/Back)
for Specific Problem Situations**

At this level, the child does not have to represent all quantities in the problem concretely. She or he has learned that a number names a quantity, that is, that a number can be stated rather than represented concretely. These strategies will develop intuitively over time. If a

child is not able to make sense of counting strategies, the child is not developmentally ready to use them and needs many more experiences with modeling solutions.

JRU Problem

Taiwan had 3 cookies. Bear gave him 5 more cookies.
How many cookies does Taiwan have now?

Counting-On-From-First Solution:

The counting sequence begins with three and continues on for five more counts.
The answer is the last number in the counting sequence.

JRU Problem

Taiwan had 3 cookies. Bear gave him 5 more cookies.
How many cookies does Taiwan have now?

Counting-On-From-Larger Solution:

The counting sequence begins with “five” and continues on for three more counts.
The answer is the last number in the counting sequence.

JCU Problem

Bear had 3 cookies. Taiwan gave him some more cookies. Now Bear has 8 cookies.
How many cookies did Taiwan give him?

Counting-On-To Solution:

A forward counting sequence starts from “four” and continues until “eight” is reached (keeping track on fingers or tally marks). The answer is the number of counting words in the sequence represented by the number of fingers or tally marks.

SRU Problem

Taiwan had 8 cookies. He gave 3 to Bear.
How many cookies does Taiwan have now?

Counting-Down Solution:

A backward counting sequence is initiated from eight. The sequence continues for three counts . . . eight, seven, six . . . the answer is the next number.

SCU Problem

Bear had 8 cookies. She gave some to Taiwan.
Now she has 3 cookies.
How many cookies did she give to Taiwan?

Counting Down To Solution:

A backward counting sequence starts from “eight” and continues until three is reached., eight, seven, six, five, four. . .” The answer is the total of number words in the counting sequence, not including the number word three.

Strategies Used by Level 3 Children (Deriving and Number Facts)

At this level, the child understands relationships between numbers and solves problems using number facts and derived facts, combining numbers to the problem when a specific fact is not at the recall level. An example of a child using a derived fact would be, “I know that nine plus four is thirteen because nine and one is ten and three more is thirteen.”

Solution Strategies Summary

When children begin to solve problems intuitively, they concretely represent the relationships in the problem. Over time, concrete strategies are abstracted to counting strategies, and eventually, as number facts are learned, children apply this knowledge to solve problems. This developmental approach differs from the practice of rote drill for memorization of facts. Children in traditional classrooms often are able to recite facts but lack understanding that a fact represents a relationship between quantities; they lack number sense. Children who have been allowed to progress through the stages described in this section come to understand these relationships.

Symbolic Procedures

Much of what has been discussed to this point has focused on children’s informal or intuitive problem-solving strategies. Such strategies are often very different from the standard symbolic procedures typically taught in the elementary school. Standard procedures provide powerful problem-solving tools; however, a concern is that many children merely memorize them. They never develop an understanding of the relationships among numbers within procedures. When allowed to progress through the stages described in the preceding section, a child will develop the habit of looking for numerical relationships. When introduced to the standard procedure, this child will understand the numerical relationships and will view the procedure simply as another strategy for solving problems. For this reason, intuitive strategies are emphasized throughout the book, *Lost and Found and Found Again*, and the game, *Taiwan’s Math Path*.

***Note to the Reader**

Cognitively Guided Instruction (CGI) is a professional development program in elementary mathematics based on an integrated program of research that focuses on the development of students’ mathematical thinking, instruction that develops that thinking, and teachers’ knowledge and beliefs about student thinking that influence how they teach. In recognition of the breadth and richness of CGI, the editors of this volume strongly recommend teachers who are interested in the approach to participate in a CGI workshop and access the resources listed below.

Cognitively Guided Instruction Resource Information

To schedule a Cognitively Guided Instruction workshop, contact Linda Levi at:
Wisconsin Center for Education Research
Cognitively Guided Instruction Suite
Madison, WI 53706
Telephone (920) 263-4267.

A text describing this approach, Children's Mathematics: Cognitively Guided Instruction, is available through the Heinemann web site: <http://www.heinemann.com>

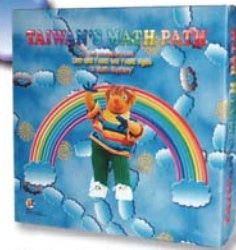
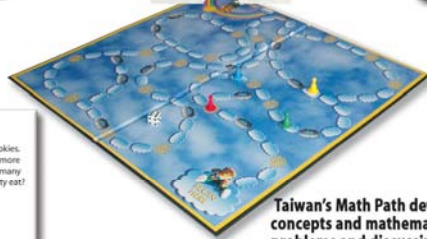
CGI web site: <http://www.wcer.wisc.edu/ncisla/publications/features/DOBconf.pdf>

References

- Brooks, J. G., & Brooks, M. G. (1993). *In search of understanding: The case for constructivist classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Carey, D. A., Fennema, E., Carpenter, T. P., & Franke, M. L. (1993). *Cognitively guided instruction: Towards equitable classrooms*. In W. Secada, E. Fennema, & L. Byrd (Eds.). *New directions in equity for mathematics education*. New York: Teacher College Press.
- Carpenter, T. P., & Fennema, E. (1992). *Cognitively guided instruction: Building on the knowledge of students and teachers*. In W. Secada (Ed.), *Curriculum reform: The case of mathematics in the United States*. Special issue of the *International Journal of Educational Research* (pp. 457–470). Elmswood, NY: Pergamon Press, Inc.
- Carpenter, T.P., Fennema, E. Franke, M.L., Levi, L., and Empson, S.B. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heineman.Collins, A.,
- Brown, J. S., & Newman, S. (1989). *Cognitive apprenticeships: Teaching the craft of reading, writing, and mathematics*. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser*. Hillsdale, NJ: Erlbaum.
- Fennema, E., Carpenter, T. P., Levi, L., Franke, M. L., & Empson, S. (1997). *Cognitively guided instruction: Professional development in primary mathematics*. Wisconsin, Madison: Wisconsin Center for Education Research.
- National Council of Teachers of Mathematics. (1998). *Teaching standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

TAIWAN'S MATH PATH[®]

characters from the book
Lost and Found and Found Again
"A Math Mystery"



There are 12 chocolate chip cookies and 8 peanut butter cookies on a plate. How many fewer peanut butter cookies are there than chocolate chip?

Level I

Rusty ate 5 cookies. Then he ate 7 more cookies. How many cookies did Rusty eat?

Level II

If Bear walks 2 miles in one hour, how many miles will he walk in 3 1/2 hours?

Fractions



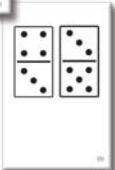
Coins

Professor Hankes baked 32 cookies. Bear ate 12 cookies and Taiwan ate 14 cookies. How many cookies were not eaten?

Level III

$20 - \underline{\quad} = (3 \times 3) \times 2$

Algebra



Domino



I make the world a better place. All players advance one space.

Sun & Clouds

A GUIDE TO HELP TEACHERS UNDERSTAND COGNITIVELY GUIDED INSTRUCTION

Teacher's Guide

Taiwan's Math Path develops understanding of important math concepts and mathematical reasoning ability. Solving math word problems and discussing problem solutions also improves reading comprehension and communication skills. Additionally, Sunshine Cards build self-esteem through positive messages of encouragement and cooperation.

Taiwan's Cookie Problems Chart - This 18" x 24" colored chart is organized to distinguish word problems in terms of difficulty. The chart is coded to identify easy, more challenging, and most challenging problems. The chart is organized according to Cognitively Guided Instruction problem types (Carpenter et al., 1999).



Wild Card



Scalene Triangle

GeoMatch Game



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http://www.ritesandrightrep.com
Developed by Math Educators,
Judith Hankes, Ph.D. and Gerald Fast, Ph.D.

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<http://www.ritesandrightrep.com/Pages/TaiwanResources.htm>
or email hankes@uwosh.edu